On the primitivity and satisfiability of SPARQL

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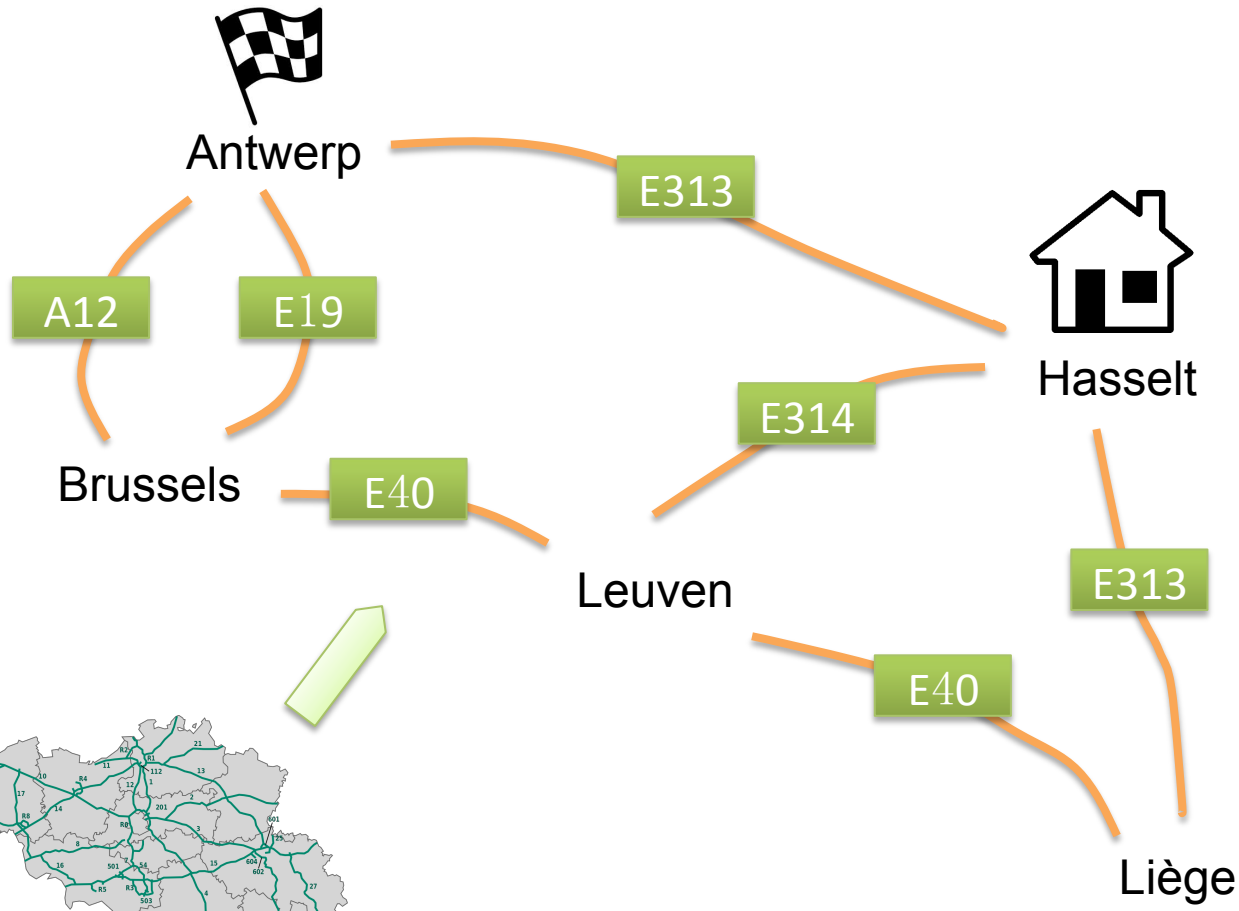
KNOWLEDGE IN ACTION

Navigating plan to DBDBD-14



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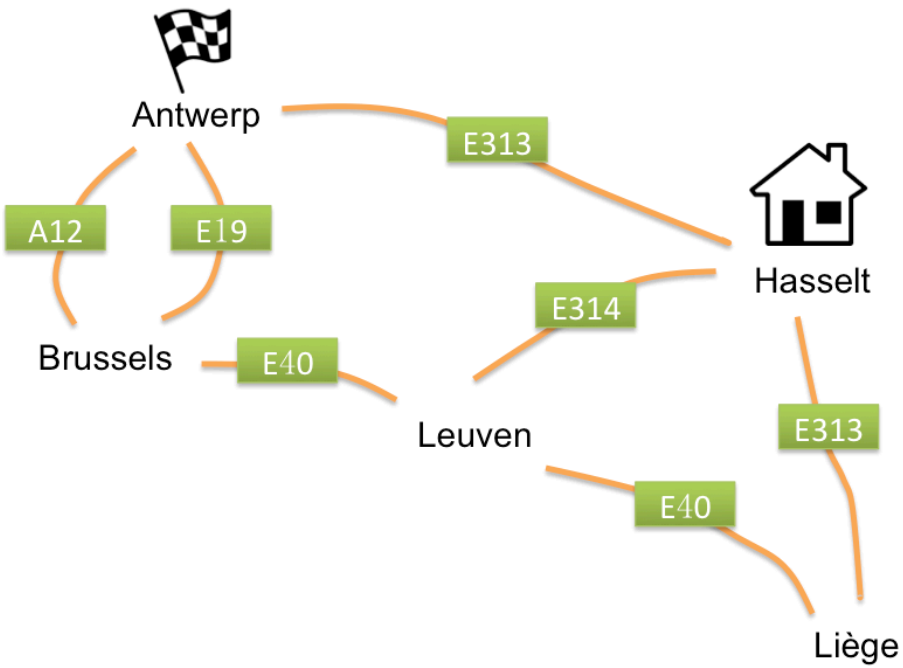
Modeling in RDF graphs

Map



Triples

(subject, predicate, object)

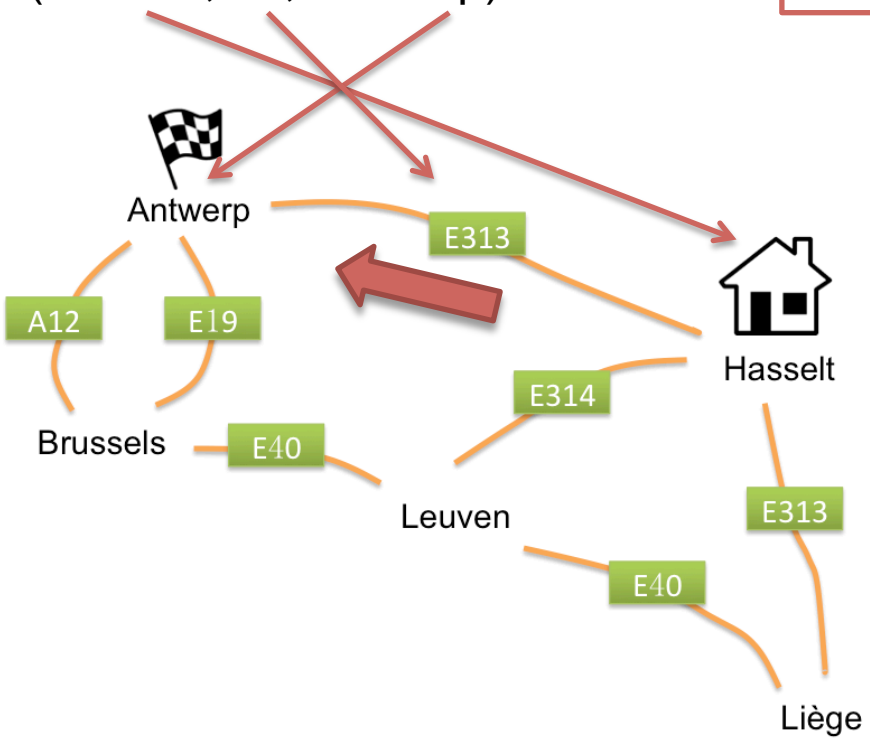
- (Hasselt, E313, Antwerp)
- (Antwerp, E313, Hasselt)
- (Hasselt, E313, Liège)
- (Liège, E313, Hasselt)
- (Hasselt, E314, Leuven)
- (Leuven, E314, Hasselt)
- (Brussels, E19, Antwerp)
- (Antwerp, E19, Brussels)
- (Brussels, A12, Antwerp)
- (Antwerp, A12, Brussels)
- (Leuven, E40, Brussels)
- (Brussels, E40, Leuven)
- (Liège, E40, Leuven)
- (Leuven, E40, Liège)

SPARQL for querying RDF graphs

- Triple patterns: a triple (u, v, w) with variables ?x, ?y, ?z…
- Pattern operators: AND, UNION, OPT, SELECT, FILTER

Which highway connects Hasselt to Antwerp?

Query: (Hasselt, ?n, Antwerp)? *Answer*: ?n = E313.



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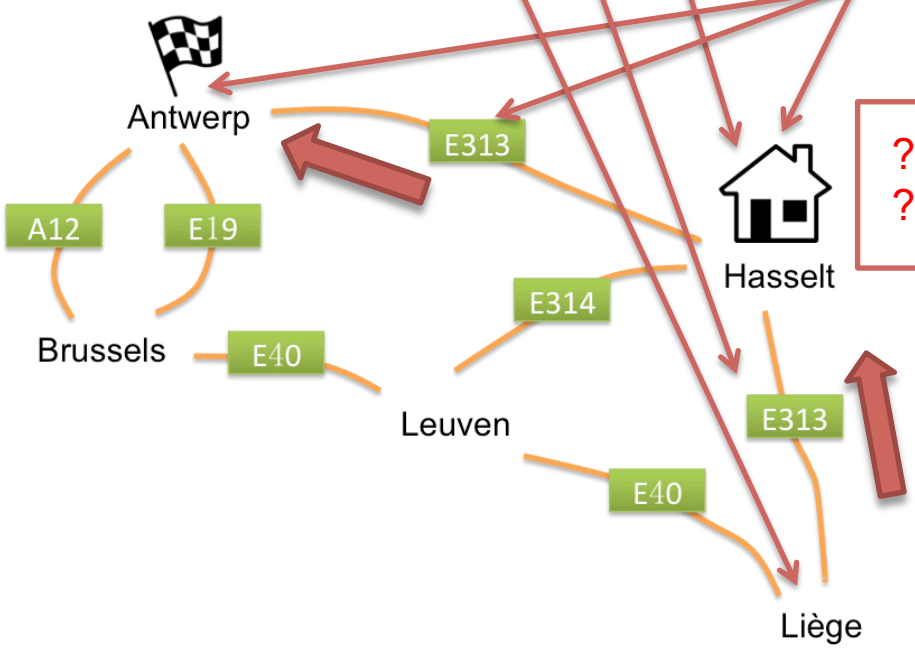
SPARQL for querying RDF graphs

- Triple patterns: a triple (u, v, w) with variables ?x, ?y, ?z…
- Pattern operators: AND, UNION, OPT, SELECT, FILTER

Which city connects to Antwerp through a city?

Query: $\text{SELECT}_{\{?x\}}((?x, ?n, ?y) \text{ AND } (?y, ?m, \text{Antwerp}))$



Answer:

?x =Liège, ?n = E313, ?m =E313, ?y = Hasselt.
?x =Leuven, ?n = E40, ?m =E19, ?y = Brussels.

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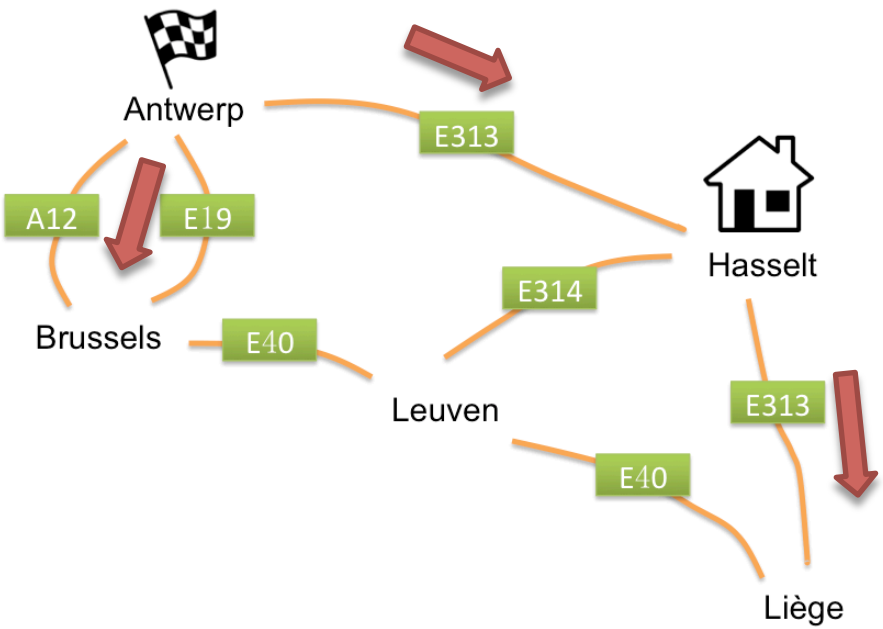
SPARQL for querying RDF graphs

- Triple patterns: a triple (u, v, w) with variables ?x, ?y, ?z…
- Pattern operators: AND, UNION, OPT, SELECT, FILTER

For each highway leaving from Antwerp, give cities reachable by the same highway in two steps. If the highway does not continue, give only the first step.?

Query:

$\text{SELECT}_{\{?x,\ ?y\}}((\text{Antwerp}, ?n, ?x)\ \text{OPT}\ (?x, ?n, ?y))\ \text{FILTER}\ (?y \neq \text{Antwerp})$



Answer:

?x =Brussels, ?n = E19.
?x =Brussels, ?n = A12.
?x =Hasselt, ?n = E313, ?y = Liège.

Outline

- Primitivity of SPARQL operators
- Satisfiability of SPARQL fragments
- Summary

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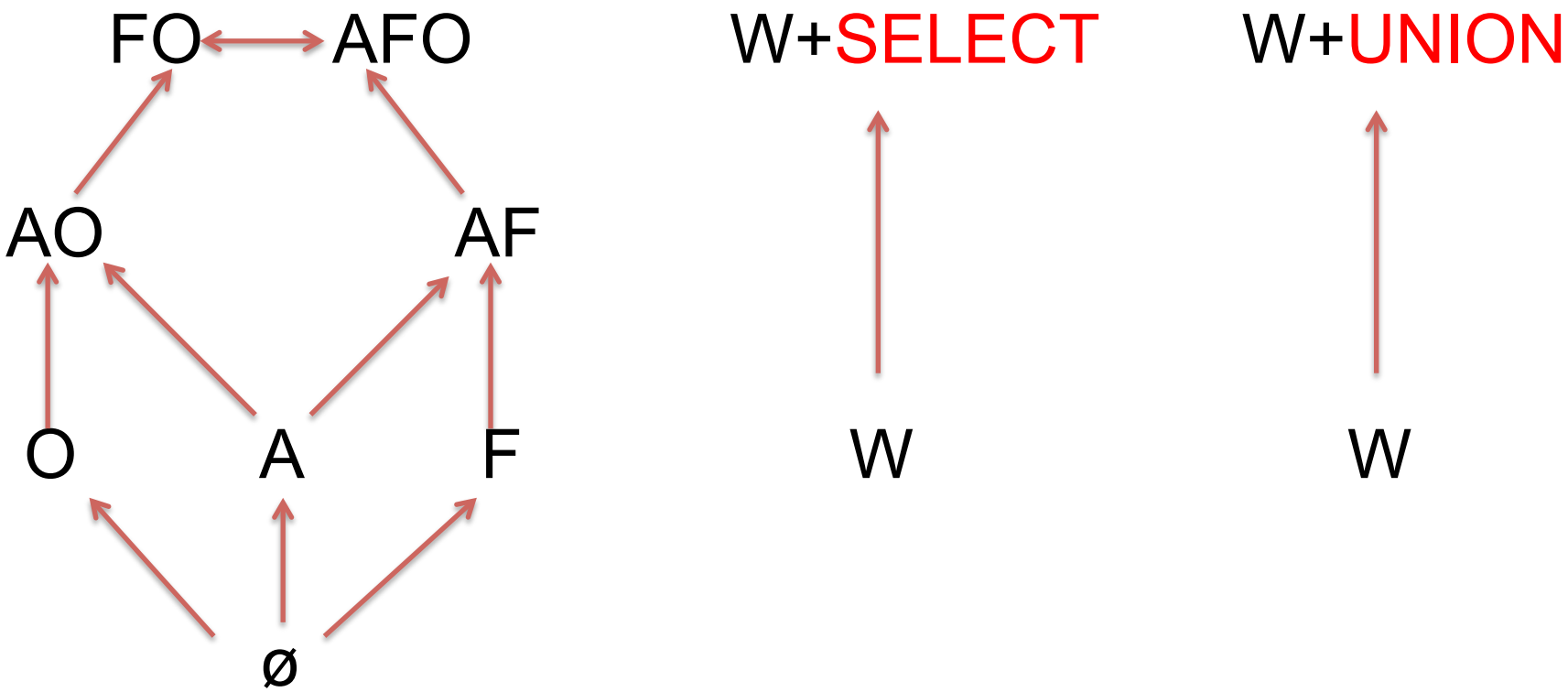
Primitivity of SPARQL operators

A fragment W1 is *expressible* in a fragment W2 if for any pattern P in W1 there exists some pattern Q in W2 such that P is equivalent to Q.

W1 → W2

Result 1



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Primitivity of SPARQL operators

AFO → FO

P AND Q = (P OPT Q) MINUS (P MINUS Q)

P MINUS Q = (P OPT (Q OPT (?x, ?y, ?z)) FILTER ¬bound(?x),

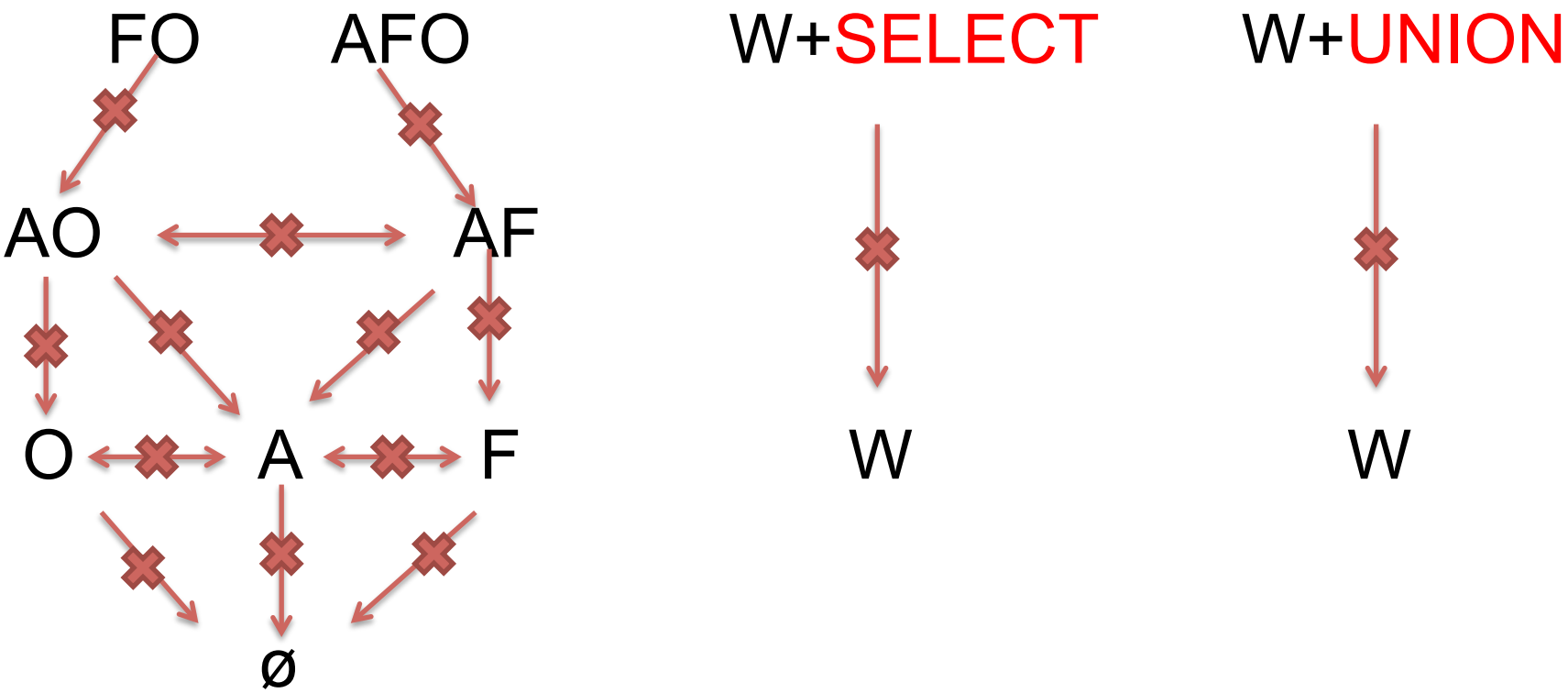
where ?x, ?y, ?z are fresh variables.

R. Angles & C. Gutierrez, 2008

Primitivity of SPARQL operators

A fragment W1 is not *expressible* in a fragment W2 if there exists some pattern P in W1 such that for any pattern Q in W2, P is not equivalent to Q.

Non-trivial results



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Primitivity of SPARQL operators

A $\not\longrightarrow$ FSU

Consider the pattern P = (?x, ?y, ?z) AND (?u, ?v, ?w): each mapping contains 6 variables. However, for any pattern Q in FSU, each mapping contains at most 3 variables.

A $\not\longrightarrow$ OSU

Consider the pattern P= (?x, p, ?y) AND (?x, q, ?y): not satisfied in any singleton graph. However, for any satisfiable pattern Q in OSU, there must be a singleton graph satisfying it.

Both OPT and FILTER are necessary to express AND.

Primitivity of SPARQL operators

An operator X is *primitive* if for any X-free fragment W, W+X is not expressible in W.

Result 2

FILTER is primitive.

$$F \not\longrightarrow AOSU$$

Consider the pattern P = (?x, ?y, ?z) FILTER ?x = ?y and the graph G = {a, b} × {a, b} × {a, b}: each mapping satisfies ?x = ?y. However, for any pattern Q in AOSU, it always contains all possible mappings from variables in Q to {a, b}.

Primitivity of SPARQL operators

Result 3

SELECT is primitive.

$$S \not\longrightarrow AOFU$$

Consider the pattern P = $SELECT_{\{?x\}}$ (?x, ?y, ?z). If there exists some pattern Q in AOFU such that P is expressible as Q. Then all variables in all triple patterns of Q are ?x. Consider the graph G = {(a, p, b)}, where a, p, b are fresh constants. P returns a mapping ?x= a, however, Q returns nothing.

Primitivity of SPARQL operators

Result 4

UNION is primitive.

U $\not\longrightarrow$ AOFS

Consider the pattern P= (?x, r, ?y) UNION (?y, r, ?x) and G = {(a, r, b)}, where there are two mappings be to returned.
However, for any pattern Q in AOFS, at most one mapping will be returned.

Primitivity of SPARQL operators

Result 5

Only **AND** is **not** primitive.

(AND, FILTER, OPT, SELECT, UNION)

Primitivity of SPARQL operators

Well-designed patterns

defined by *Perez, Arenas and Gutierrez, 2008*.

$$P_1 \text{ UNION } P_2 \text{ UNION } \ldots \text{ UNION } P_m$$

where each P_i is

- UNION-free
- Safe
- For every sub-pattern Q of the form Q_1 OPT Q_2, all variables occurring in both Q_2 and outside of Q must occur in Q_1.

P AND Q = (P OPT Q) MINUS (P MINUS Q)

P MINUS Q = (P OPT (Q OPT (?x, ?y, ?z)) FILTER ¬bound(?x).

Non-well-designed

Result 6

AND is primitive for well-designed patterns.

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Satisfiability of SPARQL fragments

A pattern P is satisfiable if there exists some graph G where P returns at least one solution.

The satisfiability problem of the full fragment (AFOSU) is undecidable because the full fragment has the same expressiveness as relational algebra whose satisfiability problem is well known to be undecidable.

Fragments via Constraints	
SPARQL(bound)	bound(?x)
SPARQL(¬bound)	¬bound(?x)
SPARQL($=_c$)	?x = c
SPARQL($\neq_c$)	?x ≠ c
SPARQL(=)	?x = ?y
SPARQL(≠)	?x ≠ ?y

Satisfiability of SPARQL fragments

Result 7

The satisfiability problem of : AFOSU(bound, =, $\neq_c$)
AFOSU(bound, $\neq$, $\neq_c$)
is decidable.

Consistency of constraints

For any set S of constraints of forms:

- bound(?x), ?x =?y, ?x ≠ c)
- bound(?x), ?x ≠ ?y, ?x ≠ c)

there always exists some mapping that satisfies each constraints of S.

Satisfiability of SPARQL fragments

Result 8

The satisfiability problem of :

AFOSU(¬bound)
AFOSU(=, ≠)
AFOSU($=_c$)

is undecidable.

The set difference operator can be expressed in

- AFOSU(¬bound)
- AFOSU(=, ≠)
- AFOSU($=_c$)

The satisfiability problem of the downward algebra (DA: union, composition, and difference) is undecidable. *T. Tan, J. Van den Bussche, X. Zhang 2014*

Satisfiability of SPARQL fragments

Result 5

The satisfiability problem of well-designed patterns is decidable.

- Every UNION-free well-designed pattern can be reduced to a pattern in AF with preserving satisfiability.
- The satisfiabilily problem of AF is decidable.

Outline

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Summary

- Presenting the Hasse diagram of expressivity of all fragments with basic SPARQL operators.
- Investigating the satisfiability problem of of all fragments with basic SPARQL operators.
- Future work: adding SPARQL 1.1 new operators, such as BIND, VALUE, MINUS etc.

Q&A

Thanks

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SPARQL for querying RDF graphs

- Triple patterns: a triple (u, v, w) with variables ?x, ?y, ?z…
- Pattern operators: AND, UNION, OPT, SELECT, FILTER

Constraints and its three-valued semantics (*true*, *false*, *error*)

- bound(?x)
- ¬ bound(?x)
- ?x = c
- ?x ≠ c
- ?x = ?y
- ?x ≠ ?y
- … ...
- disjunction: ∨
- conjunction: ∧

- μ(bound(?x)) = *true* if ?x occurs in μ; *false* otherwise
- μ(¬ bound(?x)) = *true* if ?x occurs in μ; *false* otherwise
- μ(?x = c) = *error* if ?x does not occur in μ;
- μ(?x = c) = *false* else if μ(?x) = c;
- μ(?x = c) = *true* otherwise.
- μ(?x ≠ c) = *error* if ?x does not occur in μ;
- μ(?x ≠ c) = *true* else if μ(?x) ≠ c;
- μ(?x ≠ c) = *false* otherwise.
- … …

Difference is expressible in three fragments.

Difference ($e_1 - e_2$) is expressible in SPARQL(¬bound)

$$(P_{e_1} \text{ OPT } (P_{e_2} \text{ AND } (?u, r, ?w))) \text{ FILTER } \neg\text{bound}(?u).$$

Difference ($e_1 - e_2$) is expressible in SPARQL (=, ≠)

$$\Big((P_{e_1} \text{ OPT } ((P_{e_2} \text{ AND } adom_{?u} \text{ AND } adom_{?u'}) \text{ FILTER } ?u \neq ?u')) \text{ AND } adom_{?u} \text{ AND } adom_{?u'}\Big) \text{ FILTER } ?u = ?u'.$$

Difference ($e_1 - e_2$) is expressible in SPARQL (≠, $\neq_c$)

$$\Big((P_{e_1} \text{ OPT } ((P_{e_2} \text{ AND } adom_{?u}) \text{ FILTER } ?u = a)) \text{ AND } adom_{?u}\Big) \text{ FILTER } ?u = b.$$